Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 17 May 2024

Afternoon

Paper reference **8FM0/24**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
24: Further Statistics 2
(Part of option G only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from statistical tables should be quoted in full. If a calculator is used instead of tables the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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F:1/1/1/

P 7 5 6 7 4 A 0 1 1 6

1. A continuous random variable X has cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{5}(x+1)^2 & -1 \leqslant x \leqslant 0 \\ 1 - \frac{1}{20}(4-x)^2 & 0 < x \leqslant 4 \\ 1 & x > 4 \end{cases}$$

(a) Find the probability density function, $f(x)$ **(2)**

(b) (i) Sketch $f(x)$ **(2)**

(ii) Hence describe the skewness of the distribution. **(1)**

(c) Find, to 3 significant figures, the value of c such that

$$P(1 < X < c) = P(c < X < 2)$$

(3)

(a) Differentiate each term of $F(x)$ to find $f(x)$:

$$\frac{d}{dx}\left[\frac{1}{5}(x+1)^2\right] = 2 \times \frac{1}{5} \times 1 \times (x+1)^{2-1} = \frac{2}{5}(x+1) \quad ①$$

chain rule

$$\frac{d}{dx}\left[1 - \frac{1}{20}(4-x)^2\right] = 2 \times \frac{1}{20} \times -1 \times (4-x)^{2-1} = \frac{1}{10}(4-x)$$

$$f(x) = \begin{cases} \frac{2}{5}(x+1) & -1 \leq x \leq 0 \\ \frac{1}{10}(4-x) & 0 < x \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad ①$$

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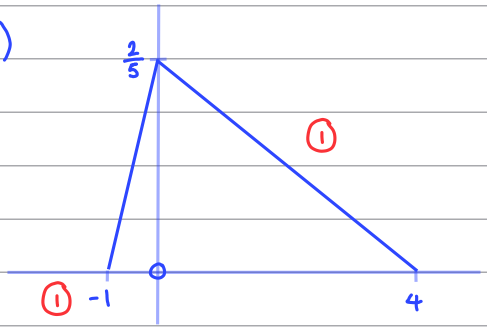
Question 1 continued

(b)(i)



$-1 \le x \le 0$ $\quad \frac{4}{10} - \frac{x}{10} = 0 \Rightarrow x = -1$

intercept $= \frac{2}{5}$ negative gradient

$0 < x \le 4$ $\quad \frac{2}{5}x + \frac{2}{5} = 0 \Rightarrow x = 4$

intercept $= \frac{2}{5}$ positive gradient

(b)(ii) Function has a longer tail to the right / positive direction so there is positive skew. 1

(c) $P(1 < X < c) = P(c < X < 2)$

$F(c) - F(1) = F(2) - F(c)$

$2F(c) = F(2) + F(1)$ 1

$2F(c) = \left[1 - \frac{1}{20}(4-2)^2\right] + \left[1 - \frac{1}{20}(4-1)^2\right]$

$2F(c) = \frac{4}{5} + \frac{11}{20}$

$2F(c) = \frac{27}{20}$

$F(c) = 0.675$

$0.675 = 1 - \frac{1}{20}(4-c)^2$

$\frac{27}{2} = 20 - 16 + 8c - c^2$

$c^2 - 8c + \frac{19}{2} = 0$

$c = \dfrac{8 \pm \sqrt{8^2 - 4 \times 1 \times \frac{19}{2}}}{2 \times 1} = \dfrac{8 - \sqrt{26}}{2}$ 1 $\leftarrow$ $1 < c < 2$ so reject other option

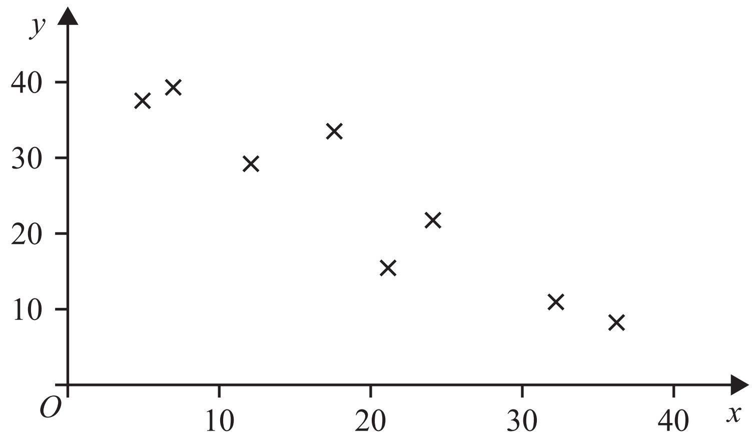
P 7 5 6 7 4 A 0 3 1 6

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. A random sample of size $n = 8$ of paired data is taken from a population. The data are plotted below.



Test, at the 1% level of significance, whether or not there is evidence of a negative **rank** correlation between the two variables.

You should state your hypotheses and critical value and show your working clearly.

(7)

$H_0 : \rho_s = 0$ $H_1 : \rho_s < 0$ ∴ one-tailed 1

Rank each x-value from 1 to 8, where 8 is the highest value

x	1	2	3	4	5	6	7	8
y	7	8	5	6	3	4	2	1
d^2	36	36	4	4	4	4	25	49

1 ← Then use the same ranking for y-values.

1

$(y-x)^2$ is squared difference

$r_s = \frac{6\Sigma d^2}{n(n^2-1)}$ ← calculate Spearman's Rank Correlation Coefficient

$r_s = \frac{6 \times (36+36+4+4+4+4+25+49)}{8(8^2-1)}$ 1

$r_s = -0.9285$ 1

At 1% significance, one-tailed test and $n = 8$,

critical value $= -0.8333$ 1 ← look up in statistical tables.

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Question 2 continued

-0.9285 is more extreme than -0.8333, so reject H_0

There is sufficient evidence to suggest that the data shows negative rank correlation. 1

(Total for Question 2 is 7 marks)

3. The continuous random variable Y has probability density function

$$f(y) = \begin{cases} \dfrac{1}{24}(y+2)(4-y) & 0 \leqslant y \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the mode of Y is 1, justifying your reasoning.

(2)

Given that $P(Y < 1) = \dfrac{13}{36}$

(b) determine whether the median of Y is less than, equal to, or greater than 2
Give a reason for your answer.

(2)

Given that $E(Y^2) = \dfrac{213}{80}$

(c) find, using algebraic integration, $\operatorname{Var}(2Y)$

(5)

(a) Mode is at maximum of $f(y)$

$f(y) = \frac{1}{24}(y+2)(4-y) = \frac{1}{24}(8 + 2y - y^2)$

$f'(y) = \frac{1}{24}(2-2y)$ (1)

At max. $\frac{1}{24}(2-2y) = 0$

$2 - 2y = 0$

$2 = 2y$

$1 = y$ ∴ Mode at $y = 1$ (1)

(b) Probability density function is symmetrical around the mode (1)

So $P(Y<2) = 2 \times P(Y<1) = 2 \times \frac{13}{36} = \frac{13}{18}$ (1)

Median is less than 2 since $\frac{13}{18} > \frac{1}{2}$. (1)

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Question 3 continued

(c) $E(Y) = \int_a^b y f(y)\, dy$, $a \le Y \le b$

$E(Y) = \int_0^3 \frac{1}{24} y\left[(y+2)(4-y)\right] dy$ ①

$E(Y) = \frac{1}{24}\int_0^3 8y + 2y^2 - y^3 \; dy$

$E(Y) = \frac{1}{24}\left[4y^2 + \frac{2}{3}y^3 - \frac{1}{4}y^4\right]_0^3$

$E(Y) = \frac{1}{24}\left[4(3)^2 + \frac{2}{3}(3)^3 - \frac{1}{4}(3)^4\right] - [0]$

$E(Y) = \frac{45}{32}$ ①

$Var(Y) = E(Y^2) - [E(Y)]^2 = \frac{213}{80} - \left(\frac{45}{32}\right)^2 = \frac{3507}{5120}$ ①

$Var(2Y) = 2^2(Var(Y))$ ①

$Var(2Y) = 4 \times \frac{3507}{5120}$

$Var(2Y) = \frac{3507}{1280} = 2.74$ ①

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4. The continuous random variable X is uniformly distributed over the interval [2, 7]

(a) Write down the value of E(X)

(1)

(b) Find $P(1 < X < 4)$

(1)

(c) Find $P(2X^2 - 15X + 27 > 0)$

(3)

(d) Find $E\left(\frac{3}{X^2}\right)$

(3)

(a) $E(X) = \frac{7+2}{2} = 4.5$ 1

uniform so expected value is the midpoint

(b) $P(1 < X < 4) = P(2 < X < 4)$ ← 1 is not in the range [2,7]

$= \frac{4-2}{7-2}$

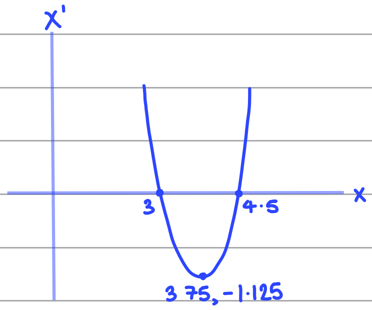
$= \frac{2}{5}$ 1

(c) $2x^2 - 15x + 27 > 0$

$(2x-9)(x-3) > 0$

Critical values are $x = 3$, $x = 4.5$

Sketch function to determine direction of inequalities.



Turning Point. $\frac{d}{dx} = 4x - 15 = 0$

$x = 3.75$

$x' = -1.125$

Question 4 continued

$\therefore \left[\{X<3\} \cup \{X>4.5\}\right]$ 1

$P(\{X<3\} \cup \{X>4.5\}) = \frac{3-2}{7-2} + \frac{7-4.5}{7-2}$ 1

$= \frac{7}{10}$ 1

(d) $E\left(\frac{3}{X^2}\right) = \int_2^7 \frac{3}{x^2} \times \frac{1}{(7-2)} \, dx$ 1

$E\left(\frac{3}{X^2}\right) = \frac{3}{5} \int_2^7 x^{-2} \, dx$

$E\left(\frac{3}{X^2}\right) = \frac{3}{5} \left[-x^{-1}\right]_2^7$ 1

$E\left(\frac{3}{X^2}\right) = \frac{3}{5} \left[-7^{-1}\right] - \left[-2^{-1}\right]$

$E\left(\frac{3}{X^2}\right) = \frac{3}{14}$ 1

(Total for Question 4 is 8 marks)

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5. A random sample of 24 adults is taken. The height, h metres, and the arm span, s metres, for each adult are recorded.

These data are summarised below.

$$S_{hh} = 0.377 \qquad S_{sh} = 0.352 \qquad \bar{s} = 1.70 \qquad \bar{h} = 1.68$$

The least squares regression line of h on s is

$$h = a + 0.919s$$

where a is a constant.

(a) Calculate the product moment correlation coefficient.

(3)

A doctor uses the least squares regression line of h on s as a model to predict a person's height based on their arm span.

(b) Use the model to predict the height of an adult with arm span 1.79 metres.

(2)

Ewan has an arm span of 1.70 metres and a height of 1.75 metres. His information is added to the sample as the 25th adult.

(c) Explain how the gradient of the regression line for the sample of 25 adults compares with the gradient of the regression line for the original sample of 24 adults.
Give a reason for your answer.

(3)

(a) $S_{ss} = \dfrac{S_{sh}}{b}$

$S_{sh} = \text{Covariance}(s,h)$
$= \beta \times \text{Var}(s)$
$= \beta \times S_{ss}$
$\therefore S_{ss} = \dfrac{S_{sh}}{\beta}$ where $h = \alpha + \beta s$

$S_{ss} = \dfrac{0.352}{0.919}$

$S_{ss} = 0.383$ ①

$r = \dfrac{S_{sh}}{\sqrt{S_{ss} \times S_{hh}}} = \dfrac{0.352}{\sqrt{0.377 \times 0.383}}$ ① $= 0.926$ ①

(b) $h - 1.68 = 0.919(1.79 - 1.70)$ ①

$h = 1.76271$ ①

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Question 5 continued

(c) S_{ss} remains the same since $s_{25} = \bar{s}$ 1

S_{sh} remains the same additional $(s-\bar{s})(h-\bar{h}) = 0$. 1

So the gradient would not change 1

Question 5 continued

(Total for Question 5 is 8 marks)

TOTAL FOR FURTHER STATISTICS 2 IS 40 MARKS